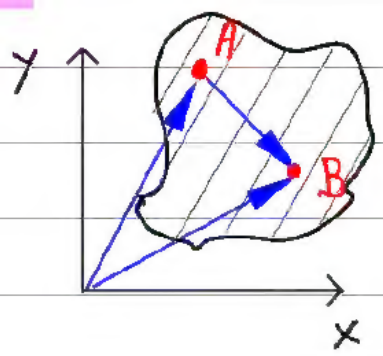
Meriam 5/1 : 2D Kinematics – Translation



Consider a 2D body in the x-y plane:
Select any 2 points, A & B.

$$\vec{r}_B = \vec{r}_A + \vec{r}_{B/A}$$

$\vec{r}_{B/A}$: vector pointing from A towards B

For the case of pure translation (with zero rotation),

- (a) all points move at the same velocity
- (b) the vector $\vec{r}_{B/A}$ is constant

$$\frac{d}{dt}\left(\vec{r}_B = \vec{r}_A + \vec{r}_{B/A}\right)$$

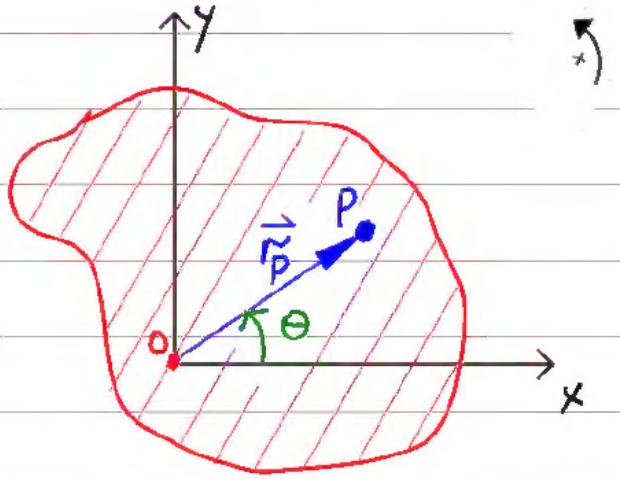
$$\rightarrow \boxed{\begin{aligned} \vec{v}_B &= \vec{v}_A + 0 \\ \vec{a}_B &= \vec{a}_A \end{aligned}}$$

Note that translation may be along a straight line or along a curvilinear path.
but orientation of body never changes.

Equations that were developed for particles in the first half of this course are generally applicable.

Meriam 5/2 : Fixed-Axis Rotation

Fixed axis at O



Consider velocity of P

Define θ to be the angle from the reference x-axis to $\vec{r}_P$ (measured counter-clockwise)

The entire body can spin with an angular velocity $\dot{\theta} = \omega$ (s^{-1})

& angular acceleration $\ddot{\theta} = \dot{\omega} = \alpha$ (s^{-2})

Note : ω and α can be $\pm$, depending on our choice of reference direction for rotation

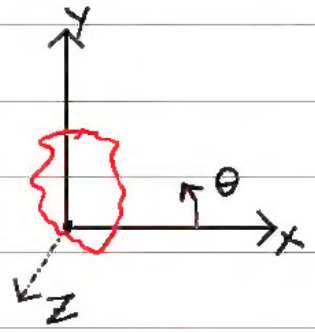
Or, you can define ω, α using 3D vector notation $+/- \hat{k}$

e.g

(a) $\omega = 5\ s^{-1}$; $\alpha = 3\ s^{-2}$

(b) $\omega = 0$; $\alpha = -2\ s^{-2}$

(c) $\vec{\omega} = -7\ \hat{k}\ s^{-1}$; $\vec{\alpha} = -2\ \hat{k}\ s^{-2}$



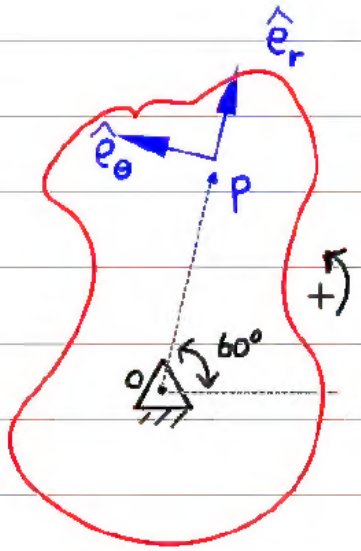
(right hand rule for rotation)

Which type of notation should you use?

Key, key concept :

A rigid body has the same values of ω, α everywhere.

→ ω and α are properties of the entire body



Find $\vec{v}_P$

use r–θ (polar) coordinates

$\omega = 3\,s^{-1}$

$\overline{OP} = 7\,m$

→ $\vec{v}_P = \dot{r}\,\hat{e}_r + r\dot{\theta}\,\hat{e}_\theta$

$\vec{v} = \omega r \hat{e}_\theta = 21\ m/s$ (150°)

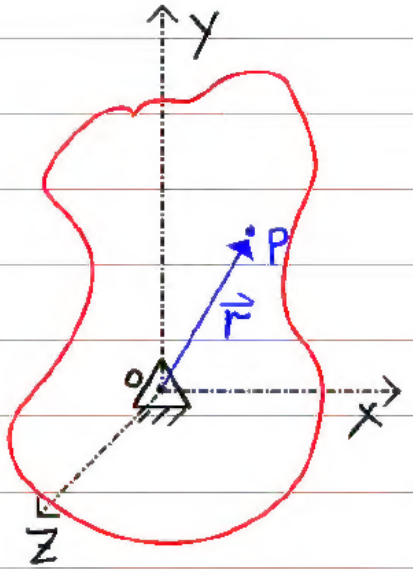
(get magnitude from $|\omega r|$; direction from diagram).

---

or use 3D vector notation:

$\vec{\omega} = 3\,\hat{k}\ s^{-1}$

$\vec{r} = 7m\,(\hat{i}\cos 60^\circ + \hat{j}\sin 60^\circ)$



$\vec{v} = \vec{\omega} \times \vec{r}$

$= (3\hat{k}) \times \left(\frac{7}{2}\hat{i} + 7\frac{\sqrt{3}}{2}\hat{j}\right)$

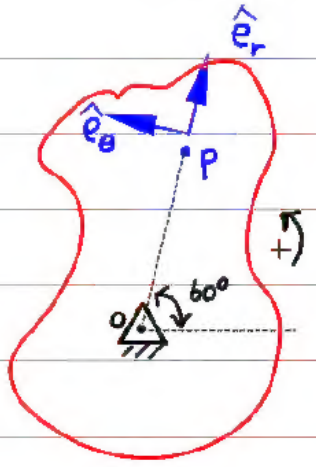
$\vec{v} = 10.5\,\hat{j} - 18.19\,\hat{i}\ \ m/s$

$v = \sqrt{10.5^2 + 18.19^2} = 21\ m/s$ (β)

$\beta = \arctan(10.5/-18.19) = 150^\circ$

Important Notes:

- $\vec{v}_P = \vec{v}_O + \vec{v}_{P/O}$ ($\vec{v}_O = 0$)
- $\vec{v}_{P/O} \perp \vec{r}_{P/O}$



→ On a solid body, any 2 points must stay the same distance away from each other.

Fixed-Axis Rotation: Acceleration

$$\vec{a}_P = \vec{a}_O + \vec{a}_{P/O}$$

($\vec{a}_O = 0$)

$$= (\ddot{r} - r\dot{\theta}^2)\hat{e}_r + (r\ddot{\theta} + 2\dot{r}\dot{\theta})\hat{e}_\theta$$

($\ddot{r} = 0$, $\dot{\theta} = \omega$, $\ddot{\theta} = \alpha$, $\dot{r} = 0$)

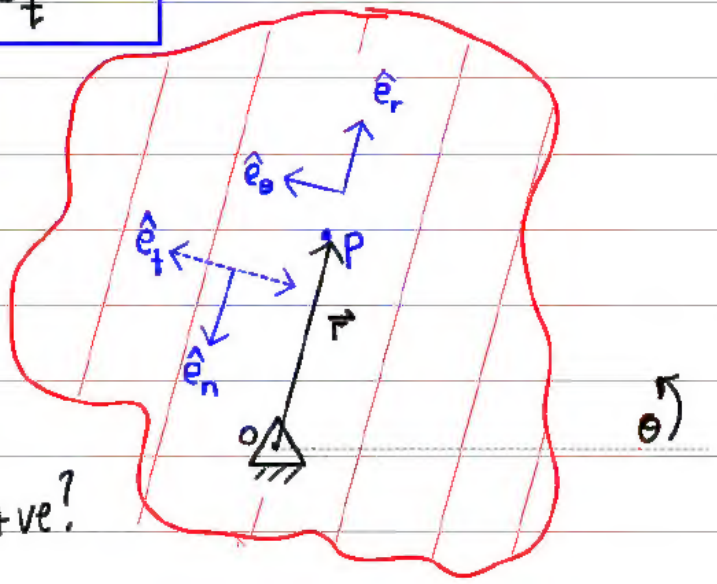
→
$$\vec{a}_P = -\omega^2 r \hat{e}_r + \alpha r \hat{e}_\theta$$

$$= \omega^2 r \hat{e}_n +/- \alpha r \hat{e}_t$$

$$= \frac{v^2}{r}\hat{e}_n + \dot{v}\hat{e}_t$$

For rotation about a fixed axis:

- $\hat{e}_n = -\hat{e}_r$
- $\hat{e}_t = +/- \hat{e}_\theta$



Suppose ω is -ve & α is +ve?

or . .

using 3D vector notation,

$$\vec{a}_p = \vec{\alpha} \times \vec{r} + \vec{\omega} \times \vec{v}$$

speeding up/slowing down component

centripetal acceleration towards O.

Note the similarity between rectilinear & rotational motion

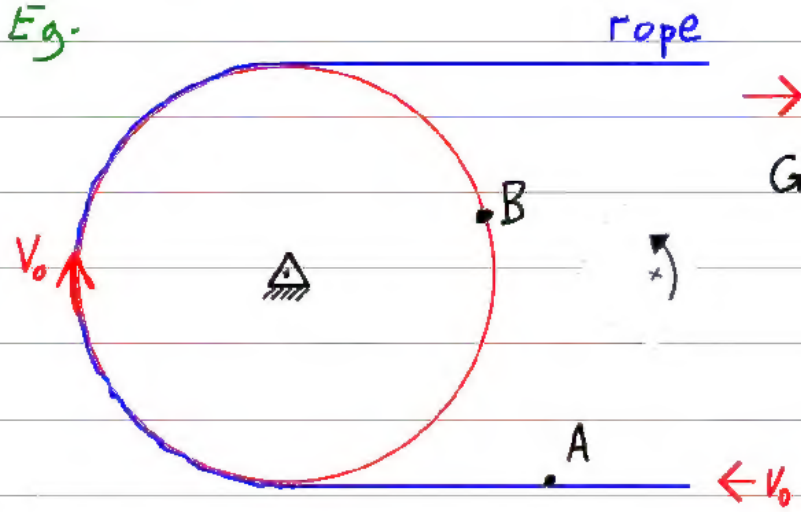
θ ⟵ position ⟶ s

$\omega = \dot{\theta}$ ⟵ velocity ⟶ $v = \dot{s}$

$\alpha = \dot{\omega}$ ⟵ acceleration ⟶ $a = \dot{v}$

Rectilinear Motion		Rotational Motion
$v = v_0 + a_0 t$	constant acceleration	$\omega = \omega_0 + \alpha_0 t$
$s = s_0 + v_0 t + \frac{1}{2} a_0 t^2$		$\theta = \theta_0 + \omega_0 t + \frac{1}{2} \alpha_0 t^2$
$v_2^2 = v_1^2 + 2 a_0 \Delta s$		$\omega_2^2 = \omega_1^2 + 2 \alpha_0 \Delta\theta$
$v\,dv = a\,ds$		$\omega\,d\omega = \alpha\,d\theta$
$\Sigma \vec{F} = \dot{\vec{G}}$		$\Sigma \vec{M}_0 = \dot{\vec{H}}_0$

Eg.



Given:

$R = 0.1$ m

$\omega_0 = -5\ s^{-1}$

$\alpha = 9\ s^{-2}$

Find $\vec{a}_A$, $\vec{a}_B$ at $t = 3$ seconds

Note:

1/ $|\vec{V}|$ and a_t are the same value everywhere on the rope.

2/ $\vec{V}$ and $\vec{a}$ of rope on rim = $\vec{V}$ and $\vec{a}$ of wheel rim

3/ $\omega(t=3) = \omega_0 + \alpha t = -5 + (9)(3) = 22\ s^{-1}$

$|a_t|$ of wheel rim $= \alpha R = (9)(0.1) = 0.9\ m/s^2$ ↑ +ve

→ a_t of A, B is $0.9\ m/s^2$

A → $\vec{a}_A = +0.9\ m/s^2\ \hat{e}_t$

$\vec{V}_A(t=3)$ →

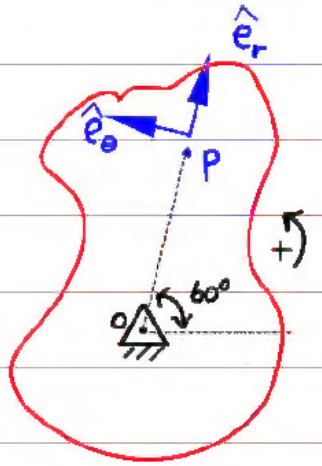
$\vec{a}_A(t=3)$ →

B Point B is turning → point B also has normal accel.

$$(a_n)_B = \frac{v^2}{\rho} = \frac{(\omega R)^2}{R} = \omega^2 R = (22)^2(0.1) = 48.4$$

$$\vec{a}_B = +0.9\,\hat{e}_t + 48.4\,\hat{e}_n \quad m/s^2$$

Summary



For **fixed axis rotation**,

- $\vec{v}_p = \omega r \, \hat{e}_\theta$
- $\vec{a}_p = \alpha r \, \hat{e}_\theta - \omega^2 r \, \hat{e}_r$
  $= +/- \alpha r \, \hat{e}_t + \omega^2 r \, \hat{e}_n$

- All points go in a circle around the fixed axis o.
- No two points get closer together or further apart.
- At any instant in time, there is only one value of ω, and one value of α, for the entire body.
- Either **n-t** or r-θ coordinates are well-suited to describe fixed-axis rotation.
  $\hat{e}_n = -\hat{e}_r$; $\hat{e}_t = +/- \hat{e}_\theta$
- Rectilinear variables s, v, a — Rotational variables θ, ω, α

Next Lecture:

- Read 5/4 (skip 5/3 for now)
- Example Problems